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SE-6031

B. E. I (Sem. II) (All) Examination

April / May – 2011

Engineering Mathematics - II

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. 1 (Sem. 2) All**

Name of the Subject :
☛ **Engineering Mathematics - II**

☛ Subject Code No. : **6 0 3 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate full marks.
(4) Draw the figure whenever it is necessary.

1 (a) Do as directed. **10**

(1) If $u=f\left(\frac{x}{y},\frac{y}{x},\frac{z}{x}\right)$ then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$.

(2) If $u=x^2-2y, v=x+y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$

(3) Find the equation of tangent plane to the surface $xyz=6$ at $(1,2,3)$.

(4) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-2)$ up to first degree.

(5) State the Taylor's interative formula to find Numerical solution of IVP

$$\frac{dy}{dx}=f(x,y), y(x_0)=y_0$$

(b) Attempt the following : **04**

(1) If $u=f(u,v)$ where $u=l\,x+my, v=ly-mx$ then prove that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\left(l^2+m^2\right)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right)$ **06**

(2) Solve the following linear system of equations by Jacobi's iteration method : **04**

$4x+y+3z=17$

$x+5y+z=14$

$2x-y+8z=12$

2 (a) Attempt any four of the following : **16**

(1) If $z(x+y)=x^2+y^2$ then prove that

$$\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$$

(2) If $\operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}(13+\tan^2 u)$$

(3) If $u=f(2x-3y,3y-4z,4z-2x)$ then prove that

$$\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$$

(4) Expand $e^x \cos y$ in powers x and y up to third degree term.

(5) The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4 cm and 6 cm respectively. The possible error in each measurement is 0.1 cm. Find approximately, the maximum possible errors in the values computed for volume and lateral surface.

(6) If $f(x,y)=0,\ \phi(y,z)=0$ then prove that

$$\frac{\partial f}{\partial y}\cdot\frac{\partial \phi}{\partial z}\cdot\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}\cdot\frac{\partial \phi}{\partial y}.$$

3 (a) Attempt any **one** of the following. **04**

(1) Find the maximum value of $u=x^p y^q z^r$ whrere x, y, z are subjected to the condition $ax+by+cz = a+b+c$.

(2) Find the extreme value of $x^3+3xy^2-3x^2-3y^2+4$

(b) Attempt any two of the following. **08**

(1) Use the method of false position to find a real root of $x^2-4x-10 = 0$ correct to three decimal places.

(2) Find a real root fo $x^2–4x–9 = 0$ correct to three decimal places using bisectio nmethod.

(3) Find a real root of $x^3 + x^2 - 1 = 0$ correct to three decimal places using interation method.

(4) Find a real root of $x\log_{10} x–1.2 = 0$, correct to three decimal places using Newton Raphson method.

(c) Attemp any one of the following : **04**

(1) The velocity V of a particle at a distance 5 from a point on its path is given by the following table

s	0	10	20	30	40	50	60
V	41	58	64	65	61	52	38

Estimate the time taken to travel 60 ft. using Simpson's $\frac{3}{8}$ – rule.

(2) Using Picard's method, solve $\frac{dy}{dx} = x + y^2, y(0) = 1$ upto second approximation.

4 (a) Do as directed. **10**

(1) Find P.I. of $(D^2+a^2)y$ - sin ax.

(2) Define the Legendre's linear differential equation with variable coefficients.

(3) Define Cauchy's nth order linear differential equation with variable coefficients.

(4) Define the following terms :

(a) Regular-singular point

(b) Ordinary point

(5) Find the general solution of $(D^3+12D^2+48D+64)y = 0$

(b) Attempt the following :

(1) Prove that $\frac{1}{f(D^2)} \cos ax = \frac{1}{f(-a)^2} \cos ax; \left(f(-a)^2 \neq 0\right)$ **04**

(2) The differential equation of the cantilever beam of length 1 is given by $\frac{d^4w}{dx^2} - \beta^4 x = 0$ if the boundary conditions are $w = \frac{dw}{dx} o \text{ for } x = 0$ and $\frac{d^2w}{dx^2} = \frac{d^3w}{dx^3} = o \text{ for } x = l$ then show that $\cos \beta l + \cosh \beta l + 1 = 0$ **06**

5 (a) Attempt the following : **06**

(1) $\left(D^2 - 5d + 6\right)y - e^2$

(2) $\left(D^2 + 2d + 1\right)y = x$

(3) $\frac{d^3 y}{cx^3} + 3\frac{d^2 y}{cx^2} + 9\frac{dy}{dx} - 27y = \cos 3x$

(b) Attempt any two of the following : **08**

(1) $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = x$

(2) $(x+3)^2 \frac{d^2 y}{dx^2} - 4(x+3)\frac{dy}{dx} + 6y = \log(x+3)$

(3) Solve $y + a^2 y = \tan ax$ - by M.V.P.

6 (a) Find the series solution of the following using Frobenius method : **10**

(1) $x(x-1)y'' + (3x-1)y' + y = 0$

(2) $2x^2 y + 3xy - \left(x^2 + 1\right)y = 0$

(3) $xy - 3y + xy = 0$

(b) Attempt any one of the following : **06**

(1) A beam of length L carries a transverse uniform load w per unit length. Find the equation of the deflection curve and maximum deflection when one end of the beam is clamped and the other is simply supported.

(2) The differential equation for a circuit in which self inductance and capacitance neutralize each other as $L\frac{d^2 i}{ct^2} + \frac{i}{c} = 0$. Find the current l as a function of t, given that i is the maximum current and i=0 when t=0.
